IU-MSTP/9; hep-th/9603159
April 1996

Path Integral Evaluation of Non-Abelian Anomaly and Pauli–Villars–Gupta Regularization

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ABSTRACT

When the path integral method of anomaly evaluation is applied to chiral gauge theories, two different types of gauge anomaly, i.e., the consistent form and the covariant form, appear depending on the regularization scheme for the Jacobian factor. We clarify the relation between the regularization scheme and the Pauli–Villars–Gupta (PVG) type Lagrangian level regularization. The conventional PVG, being non-gauge invariant for chiral gauge theories, in general corresponds to the consistent regularization scheme. The covariant regularization scheme, on the other hand, is realized by the generalized PVG Lagrangian recently proposed by Frolov and Slavnov. These correspondences are clarified by reformulating the PVG method as a regularization of the composite gauge current operator.

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It is well known that when one applies the path integral method of anomaly evaluation [1] to chiral gauge theories, two different types of gauge anomaly [2], i.e., the consistent form [3,4] and the covariant form [1,4], appear depending on the regularization scheme for the Jacobian factor. In this letter, we clarify relations between those two regularization schemes and the Pauli–Villars–Gupta (PVG) type Lagrangian level regularization. The conventional PVG [5], being non-gauge invariant in chiral gauge theories, in general corresponds to the consistent regularization scheme in [6]. The covariant regularization scheme in [1], on the other hand, is realized by the generalized PVG Lagrangian recently proposed by Frolov and Slavnov [7,8,9,10].

These correspondences will be clarified by reformulating the PVG method as a regularization of composite gauge current operators. A similar analysis on the generalized PVG regularization proposed by Narayanan and Neuberger [8] has been performed in [10]. The correspondence between the consistent scheme in the path integral method and the conventional PVG has also been noticed sometimes [6,11]. Our main concern here is the correspondence between the *covariant* scheme and the *generalized* PVG in [7] but we will present the analysis on both cases to contrast the two regularization schemes.

Let us first recapitulate the essence of the two regularization schemes in the path integral framework. The scheme is directly related to the definition of the path integral measure in the partition function: $Z=\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\exp(\int d^4x\,\overline{\psi}i\not{D}\psi)$, where the covariant derivative is defined by[†] $\not{D}\equiv\gamma^\mu(\partial_\mu-igA_\mu^aT^aP_R)$.

I. The "consistent" regularization scheme [6]: One introduces the eigenfunction of the covariant derivative and its hermite conjugate

$$\not{D}\varphi_n=\lambda_n\varphi_n,\quad \not{D}^\dagger\chi_n=\lambda_n^*\chi_n, \tag{1}$$

† Throughout this article, we work in Euclidean spacetime, $ix^0=x^4$, $A_0=iA_4$, $i\gamma^0=\gamma^4$ and $\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3=\gamma^4\gamma^1\gamma^2\gamma^3$. In particular, $\gamma^{\mu\dagger}=-\gamma^\mu$, $\gamma_5^\dagger=\gamma_5$, $g_{\mu\nu}=-\delta_{\mu\nu}$ and $\varepsilon^{1234}=1$. The chirality projection operator is defined by $P_{R,L}\equiv(1\pm\gamma_5)/2$. The anomaly in Minkowski spacetime is obtained by multiplying a factor i.

where $\not{D}^\dagger = \gamma^\mu(\partial_\mu - igA_\mu^a T^a P_L) \neq \not{D}$, and satisfies the orthonormal relation $\int d^4x\, \chi_n^\dagger(x)\varphi_m(x) = \delta_{n,m}$. The eigenvalue λ_n in (1) is *not* invariant under the chiral gauge transformation [6]. The fermion fields are then decomposed as $\psi = \sum_n a_n\varphi_n$, $\overline{\psi} = \sum_n \chi_n^\dagger \overline{b}_n$, and the path integral measure is defined by $\mathcal{D}\psi\mathcal{D}\overline{\psi} \equiv \prod_n da_n d\overline{b}_n$. By considering an infinitesimal change of variable under the gauge transformation, $\psi \to (1 + iw^a T^a P_R)\psi$, $\overline{\psi} \to \overline{\psi}(1 - iw^a T^a P_L)$, and the resultant Jacobian factor [1], one has an un-regularized anomalous Ward identity:‡

$$D_\mu \langle J^{\mu a}(x)\rangle = i\sum_n \chi_n^\dagger T^a \gamma_5 \varphi_n. \tag{2}$$

The right hand side is regularized by using the eigenvalue λ_n in (1):

$$\begin{aligned} D_\mu \langle J^{\mu a}(x)\rangle_{\text{consistent}} &\equiv \lim_{\Lambda\to\infty} i\sum_n \chi_n^\dagger T^a \gamma_5 f(\lambda_n^2/\Lambda^2)\varphi_n \\ &= \lim_{\Lambda\to\infty} i \lim_{y\to x} \operatorname{tr}\left[T^a\gamma_5 f(\not{D}^2/\Lambda^2)\delta(x-y)\right], \end{aligned} \tag{3}$$

where the completeness relation, $\sum_n \varphi_n(x)\chi_n^\dagger(y) = \delta(x-y)$ has been used. The regulator function in (3) is an arbitrary function which dumps sufficiently fast [1], $f(0) = 1$, $f(\infty) = f'(\infty) = f''(\infty) = \cdots = 0$. Then by using $\delta(x-y) = \int d^4k\, e^{ik(x-y)}/(2\pi)^4$, the gauge anomaly is evaluated as [6]

$$\begin{aligned} &D_\mu \langle J^{\mu a}(x)\rangle_{\text{consistent}} \\ &= \frac{ig^2}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma} \operatorname{tr}\left[T^a\partial_\mu(A_\nu\partial_\rho A_\sigma - \frac{ig}{2}A_\nu A_\rho A_\sigma)\right] + \lim_{\Lambda\to\infty} D_\mu \frac{\delta}{\delta(gA_\mu^a(x))}\int d^4x\, L(x) \end{aligned} \tag{4}$$

where $A_\mu = A_\mu^a T^a$ and the local functional of $A_\mu(x)$, $L(x)$ is

$$L(x) = -\frac{g^2}{16\pi^2}\Lambda^2 \int_0^\infty dt\, f(t) \operatorname{tr}(A_\mu A^\mu) + \frac{g^2}{96\pi^2}\operatorname{tr}(A_\mu \Box A^\mu) + O(\Lambda^0, A^3). \tag{5}$$

Note that the "intrinsic" part of the anomaly is independent of the regulator function $f(t)$. On the other hand, reflecting the fact that the eigenvalue λ_n in (1) does

‡ $D_\mu \langle J^{\mu a}(x)\rangle \equiv \partial_\mu \langle J^{\mu a}(x)\rangle + gf^{abc}A_\mu^b \langle J^{\mu c}(x)\rangle$, where $[T^a, T^b] = if^{abc}T^c$.

not have a gauge invariant meaning, i.e., the regularization explicitly breaks the gauge symmetry, a fake "anomaly" which is expressed as a gauge variation of a local functional $L(x)$ appears even for the anomaly free case, $\mathrm{tr}(T^a\{T^b,T^c\})=0$.

The gauge anomaly in (4) satisfies the Wess–Zumino consistency condition [3,4], thus this regularization scheme [6] gives rise to the consistent form of anomaly. Moreover when a gauge field which couples to the left handed component is introduced, the regularization scheme gives V-A (or Bardeen) form [4]; in particular there is no fermion number anomaly [12], $\partial_\mu \langle J^\mu(x)\rangle_{\text{consistent}}=0$.

II. The "covariant" regularization scheme [1]: The eigenfunction of *hermite* operator $\not{D}^\dagger\not{D}$ and $\not{D}\not{D}^\dagger$ is introduced:

$$\not{D}^\dagger\not{D}\varphi_n(x)=\lambda_n^2\varphi_n(x),\quad \not{D}\not{D}^\dagger\phi_n(x)=\lambda_n^2\phi_n(x), \tag{6}$$

where λ_n is real positive. It follows from this definition that $\not{D}\varphi_n=\lambda_n\phi_n$ and $\not{D}^\dagger\phi_n=\lambda_n\varphi_n$. They are orthonormal as $\int d^4x\,\varphi_n^\dagger(x)\varphi_m(x)=\int d^4x\,\phi_n^\dagger(x)\phi_m(x)=\delta_{n,m}$. The fermion field is then decomposed by using those eigenfunctions $\psi=\sum_n a_n\varphi_n$, $\overline{\psi}=\sum_n\phi_n^\dagger\overline{b}_n$, and the path integral measure is defined by $\mathcal{D}\psi\mathcal{D}\overline{\psi}\equiv\prod_n da_n d\overline{b}_n$.

The Jacobian factor associated with the gauge transformation now gives [1]:

$$D_\mu\langle J^{\mu a}(x)\rangle=i\sum_n\left[\varphi_n^\dagger T^aP_R\varphi_n-\phi_n^\dagger T^aP_L\phi_n\right]. \tag{7}$$

Therefore one defines the regularized anomalous Ward identity as

$$\begin{aligned}
&D_\mu\langle J^{\mu a}(x)\rangle_{\text{covariant}}\\
&\equiv\lim_{\Lambda\to\infty} i\sum_n\left[\varphi_n^\dagger T^aP_Rf(\lambda_n^2/\Lambda^2)\varphi_n-\phi_n^\dagger T^aP_Lf(\lambda_n^2/\Lambda^2)\phi_n\right]\\
&=\lim_{\Lambda\to\infty} i\lim_{y\to x}\mathrm{tr}\left[T^aP_Rf(\not{D}^\dagger\not{D}/\Lambda^2)-T^aP_Lf(\not{D}\not{D}^\dagger/\Lambda^2)\right]\delta(x-y),
\end{aligned}\tag{8}$$

which gives rise to the covariant form [1,4] of the gauge anomaly:

$$D_\mu\langle J^{\mu a}(x)\rangle_{\text{covariant}}=\frac{ig^2}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\big(T^aF_{\mu\nu}F_{\rho\sigma}\big), \tag{9}$$

where the field strength is defined by $F_{\mu\nu} \equiv \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c\right) T^a$. The fermion number anomaly [12] is also given by $\partial_\mu \langle J^\mu(x) \rangle_{\text{covariant}} = i g^2 \varepsilon^{\mu\nu\rho\sigma} \times \mathrm{tr}(F_{\mu\nu} F_{\rho\sigma})/32\pi^2$.

Since the eigenvalue λ_n in (6) is invariant under the chiral gauge transformation, the anomaly is expressed solely by the field strength, being gauge covariant. This holds even if the gauge representation is anomalous, for which there exists no gauge invariant regularization. The trick in this regularization scheme is that the gauge vertex associated with the gauge current (8) and the other gauge vertices are differently treated; it thus explicitly spoils the *Bose* symmetry among the gauge vertices [1]. Therefore other conventional regularization which automatically preserves the Bose symmetry (such as the momentum cutoff, dimensional, PVG and lattice) does not correspond to the covariant scheme *in general*. The only exception is the anomaly free case, for which the right hand side of (9) vanishes. Only in that case, there is a chance to relate the covariant scheme with other Bose symmetric regularization scheme.

Let us now consider the conventional PVG regularization [5] for the chiral fermion:

$$\mathcal{L} = \overline{\psi} i \not{D} \psi - \overline{\psi} M \psi + \overline{\phi} i \not{D} \phi - \overline{\phi} M' \phi, \tag{10}$$

where ψ and ϕ are fermionic and bosonic Dirac spinor respectively and each of which has the gauge and an internal space (generation) indices. The classical gauge current is defined by $J^{\mu a}(x) = \overline{\psi} \gamma^\mu T^a P_R \psi + \overline{\phi} \gamma^\mu T^a P_R \phi$. Since the Lagrangian is *not* invariant under the chiral gauge transformation, the gauge current does not covariantly conserve:

$$D_\mu J^{\mu a}(x) = i \overline{\psi} M T^a \gamma_5 \psi(x) + i \overline{\phi} M' T^a \gamma_5 \phi(x). \tag{11}$$

The conventional PVG regularization, as a regularization of the gauge current

composite operator, can be summarized in the following form (see also [10]):

$$\langle J^{\mu a}(x)\rangle_{\text{cPVG}} = \lim_{y\to x} \text{tr}\left[(-1)\gamma^\mu T^a P_R \left\langle T\psi(x)\overline{\psi}(y)\right\rangle + \gamma^\mu T^a P_R \left\langle T\phi(x)\overline{\phi}(y)\right\rangle\right]. \tag{12}$$

All the fermion one loop diagrams, including the contribution of the regulator fields, can be deduced by taking the functional derivative of (12) with respect to the background gauge field. Using the formal full propagator in the fixed background,

$$\left\langle T\psi(x)\overline{\psi}(y)\right\rangle = \frac{-1}{i\not{D} - M}\delta(x-y), \quad \left\langle T\phi(x)\overline{\phi}(y)\right\rangle = \frac{-1}{i\not{D} - M'}\delta(x-y), \tag{13}$$

the regularized gauge current operator (12) can be written as

$$\begin{aligned}
&\langle J^{\mu a}(x)\rangle_{\text{cPVG}} \\
&= \lim_{y\to x} \text{tr}\left[(-1)\gamma^\mu T^a P_R \frac{-1}{i\not{D} - M}\delta(x-y) + \gamma^\mu T^a P_R \frac{-1}{i\not{D} - M'}\delta(x-y)\right] \\
&= \lim_{y\to x} \text{tr}\left[\gamma^\mu T^a P_R \frac{1}{i\not{D}} \sum_n \frac{(-1)^n \not{D}^2}{\not{D}^2 + m_n^2}\delta(x-y)\right] \\
&\equiv \lim_{y\to x} \text{tr}\left[\gamma^\mu T^a P_R \frac{1}{i\not{D}} f(\not{D}^2/\Lambda^2)\delta(x-y)\right].
\end{aligned} \tag{14}$$

In deriving the second expression, we have used the fact that the trace of an odd number of gamma matrices vanishes. We have also diagonalized the mass matrices (they are hermite) and assigned even generation index for fermions and odd generation index for bosons. Finally the regulator function $f(t)$ has been defined by

$$f(t) \equiv \sum_n \frac{(-1)^n t}{t + m_n^2/\Lambda^2}. \tag{15}$$

Obviously $f(0) = 1$ and the PVG condition [5], $\sum_n (-1)^n = \sum_n (-1)^n m_n^2 = 0$, implies $f(t) = O(1/t^2)$ for $t \to \infty$. (The simplest choice is $m_0 = 0$, $m_2 = \sqrt{2}\Lambda$, $m_1 = m_3 = \Lambda$, and $f(t) = 2/(t+1)(t+2)$.) Eqs. (14) and (15) summarize the structure of the conventional PVG regularization in a neat way.

Let us evaluate the covariant divergence of the composite current operator (14):

$$
\begin{aligned}
D_\mu \langle J^{\mu a}(x)\rangle_{\rm cPVG} &= D_\mu \left[\sum_n \frac{1}{i\lambda_n} f(\lambda_n^2/\Lambda^2) \chi_n^\dagger(x) \gamma^\mu T^a P_R \varphi_n(x) \right] \\
&= -i \sum_n \frac{1}{\lambda_n} f(\lambda_n^2/\Lambda^2) \left[(-\not{D}^\dagger \chi_n)^\dagger T^a P_R \varphi_n + \chi_n^\dagger T^a P_L \not{D} \varphi_n \right] \\
&= i \sum_n \chi_n^\dagger T^a \gamma_5 f(\not{D}^2/\Lambda^2) \varphi_n .
\end{aligned}
\tag{16}
$$

Note that we have used the properties of the eigenfunction *in* (1). Comparing (16) and (3) we realize that the conventional PVG in general corresponds to the consistent regularization scheme in the path integral formulation [6,11]. This connection is also suggested by the following considerations: 1) The conventional PVG, being a Lagrangian level regularization, provides a well defined generating functional which should satisfy the Wess–Zumino consistency condition [3]. 2) The mass terms in the conventional PVG regularization (10) explicitly breaks the chiral gauge symmetry, as the non-gauge invariant eigenvalue in (1) does. 3) The conventional PVG (10) is invariant under the *vector* gauge transformation, thus one has the V-A form of the gauge anomaly, in particular, no fermion number anomaly.

We have observed that the evaluation of the non-Abelian anomaly in the conventional PVG regularization results in the calculation (4). For example, the fake anomaly part (5) indicates the vacuum polarization tensor has a non-transverse piece, $-\Lambda^2 \int dt\, f(t)\, \mathrm{tr}(T^a T^b) g^{\mu\nu}/(8\pi^2) + \cdots$, when the conventional PVG is adapted.

Another way to evaluate the gauge anomaly in the conventional PVG is to compute directly the right hand side of the classical Ward identity (11). By the same procedure as above, it is easy to see that it again gives the last line of (16), thus the same anomaly (4).

Now we question whether the covariant regularization scheme (8) and (9) can be implemented by a PVG type Lagrangian level regularization. The answer seems somewhat non-trivial: 1) The Lagrangian level regularization in general, as mentioned above, gives the consistent form of the gauge anomaly; the construction of

such a Lagrangian level covariant regularization is possible *only* for the anomaly free case. 2) The PVG type mass term should be invariant under the chiral gauge transformation, as the gauge symmetry in the external gauge vertices is preserved in (9). 3) To give the fermion number anomaly [12], the Lagrangian should explicitly breaks the associated U(1) symmetry.

We show below that the *generalized* PVG regularization proposed by Frolov and Slavnov [7] gives the answer. For simplicity, only the analysis for real and pseudo-real gauge representations (therefore is automatically anomaly free) will be presented [13].⋆

The generalized PVG Lagrangian is given by

$$\mathcal{L} = \overline{\psi} i \not{D} \psi - \frac{1}{2} \psi^T U^\dagger M C \psi + \overline{\phi} X i \not{D} \phi - \frac{1}{2} \phi^T U^\dagger M' C \phi + \text{h.c.}, \tag{17}$$

where C is the charge conjugation matrix and U is the unitary matrix such that $T^a = -U T^{a*} U^\dagger = -U T^{aT} U^\dagger$. For a real representation U is a symmetric matrix, and for a pseudo-real representation U is an anti-symmetric matrix. The salient feature of the formulation [7] is the number of the regulator fields may be infinite (see also [8,9,10]). The matrix X is introduced to avoid the tachyonic field [7] and can be taken as $X = \text{diag}(1, -1, 1, -1, \cdots)$.

Note that the PVG mass terms in (17) is Majorana type and they *are* gauge invariant. The classical gauge current consequently does conserve $D_\mu J^{\mu a}(x) = 0$ (compare with (11)). This is consistent even in the quantum level because the theory is anomaly free. The statistics of the fields requires that M' (M) is (anti-)symmetric for the pseudo-real representation, and M (M') is (anti-)symmetric

⋆ The main concern in [7] is a gauge invariant regularization for an anomaly free *complex* representation, i.e., the irreducible spinor representation of SO(10), that is important from the view point of the application to the Standard model. The generator or the gauge current of the irrep. may be decomposed as $T^a(1+\Gamma_{11})/2 = T^a/2 + T^a\Gamma_{11}/2$. The "real part" $T^a/2$ is regularized basically in the same way as the pseudo-real case. The "imaginary part" $T^a\Gamma_{11}/2$ is finite due to a special property of Γ_{11} [7]. See also [13].

for the real representation. For example, we can take

$$M = \begin{pmatrix} 0 & & & & & \\ & 0 & 2 & & & \\ & -2 & 0 & & & \\ & & & 0 & 4 & \\ & & & -4 & 0 & \\ & & & & & \ddots \end{pmatrix} \Lambda, \quad M' = \begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & & & \\ & & 0 & 3 & \\ & & 3 & 0 & \\ & & & & \ddots \end{pmatrix} \Lambda, \tag{18}$$

for the pseudo-real case, and

$$M = \begin{pmatrix} 0 & & & & & \\ & 2 & & & & \\ & & 2 & & & \\ & & & 4 & & \\ & & & & 4 & \\ & & & & & \ddots \end{pmatrix} \Lambda, \quad M' = \begin{pmatrix} 0 & 1 & & & \\ -1 & 0 & & & \\ & & 0 & 3 & \\ & & -3 & 0 & \\ & & & & \ddots \end{pmatrix} \Lambda, \tag{19}$$

for the real case. For the former case, it can be shown [13] that an infinite number of regulator fields is always needed in (17), while for the latter case, we may use a finite number of them, such as $M = \left(\begin{smallmatrix} 0 & 0 \\ 0 & \sqrt{2} \end{smallmatrix}\right)\Lambda$, $M' = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)\Lambda$.

Since the gauge current in (17) is given by $J^{\mu a}(x) = \overline{\psi}\gamma^\mu T^a P_R \psi + \overline{\phi} X \gamma^\mu T^a P_R \phi$, the regularized gauge current operator is defined by (as we have done for the conventional PVG regularization in (12)),

$$\begin{aligned} \langle J^{\mu a}(x) \rangle_{\text{gPVG}} &= \lim_{y \to x} \operatorname{tr} \left[\gamma^\mu T^a P_R \frac{1}{i \not{D}} \sum_{n=-\infty}^{\infty} \frac{(-1)^n \not{D} \not{D}^\dagger}{\not{D} \not{D}^\dagger + n^2 \Lambda^2} \delta(x-y) \right] \\ &= \lim_{y \to x} \operatorname{tr} \left[\gamma^\mu T^a P_R \frac{1}{i \not{D}} f(\not{D} \not{D}^\dagger / \Lambda^2) \delta(x-y) \right], \end{aligned} \tag{20}$$

where we have used the formal full propagator

$$\begin{aligned} \langle T \psi(x) \overline{\psi}(y) \rangle &= i \not{D}^\dagger \frac{1}{\not{D} \not{D}^\dagger + M^\dagger M} \delta(x-y), \\ \langle T \phi(x) \overline{\phi}(y) \rangle &= i \not{D}^\dagger X^{-1} \frac{1}{\not{D} \not{D}^\dagger + M'^\dagger M'} \delta(x-y). \end{aligned} \tag{21}$$

The regulator function $f(t)$ is now defined by [7]

$$f(t) \equiv \sum_{n=-\infty}^{\infty} \frac{(-1)^n t}{t+n^2} = \frac{\pi\sqrt{t}}{\sinh(\pi\sqrt{t})}. \tag{22}$$

Incidentally, the regularized form of the gauge current (20) is identical to the one in the covariant regularization in [11], which is formulated as a form factor insertion to the fermion propagator. It has also been known [11] that the regularized form (20) corresponds to the covariant scheme in [1]. The regularization furthermore can be interpreted as the gauge invariant (Euclidean) proper time cutoff [14]:

$$\frac{1}{i\not{D}} f(\not{D}\not{D}^\dagger/\Lambda^2) = -i \int_0^\infty d\tau\, g(\Lambda^2\tau) e^{-\tau \not{D}\not{D}^\dagger} \tag{23}$$

where $g(x)$ is the inverse Laplace transformation of $f(t)/t$. (For example, for $f(t)$ in (22), $g(x) = \vartheta_0(0, e^{-x})$, and for $f(t) = e^{-t}$, $g(x) = \theta(x-1)$.) It would be interesting to investigate a string theory type interpretation of the infinite tower of the PVG regulator, on the basis of the proper time representation (23).

Now let us compute the divergence of the regularized gauge current composite operator (20) to see the relation to the covariant regularization (8) (see also [11]). We first use the completeness relation of $\phi_n(x)$ *in* (6), $\sum_n \phi_n(x)\phi_n^\dagger(y) = \delta(x-y)$ in (20). Then the calculation proceeds as follows:

$$\begin{aligned} D_\mu \langle J^{\mu a}(x)\rangle_{\mathrm{gPVG}} &= D_\mu \left[\sum_n \frac{1}{i\lambda_n^2} f(\lambda_n^2/\Lambda^2)\phi_n^\dagger(x)\gamma^\mu T^a P_R \not{D}^\dagger \phi_n(x) \right] \\ &= -i \sum_n \frac{1}{\lambda_n} f(\lambda_n^2/\Lambda^2) \left[(-\not{D}^\dagger\phi_n)^\dagger T^a P_R \varphi_n + \phi_n^\dagger T^a P_L \not{D}\varphi_n \right] \\ &= i \sum_n \left[\varphi_n^\dagger T^a P_R f(\not{D}^\dagger\not{D}/\Lambda^2)\varphi_n - \phi_n^\dagger T^a P_L f(\not{D}\not{D}^\dagger/\Lambda^2)\phi_n \right]. \end{aligned} \tag{24}$$

This is identical to (8). It is also possible to verify that the correct fermion number anomaly [12] is reproduced within this formulation [13]. Therefore we realize that

the generalized PVG regularization by Frolov and Slavnov [7] corresponds to the covariant regularization scheme in the path integral formulation for the anomaly evaluation. This is our main result.

A full detail analysis on the generalized PVG [7] is reported in [13]: It can be verified the vacuum polarization tensor is transverse without any gauge variant counter terms. The non-gauge anomalies, such as the conformal anomaly, have a gauge invariant form.

We thank T. Fujiwara for enlightening discussions. The work of H.S. is supported in part by Monbusho Grant-in-Aid Scientific Research No. 07740199 and No. 07304029.

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